Moon-Shadow Eclipse Probability for High-Altitude Spacecraft

Richard R. Williams*
TRW Systems Group, Redondo Beach, Calif.

An analytical method is described for estimating the probability that a spacecraft in a nearly circular orbit about the Earth will encounter a moon-shadow eclipse. The probabilistic approach is well-suited for the early phases of a mission analysis study, because it is able to handle uncertainties about the eventual date and time of launch. Appropriate probability density functions are calculated, and the dependence of eclipse probability on orbit inclination is shown. Numerical calculations are presented for the VELA spacecraft, with particular attention being given to the possibility of nearly simultaneous eclipses caused by the Earth and the moon. It has been found that the probability of the latter event is approximately 0.040 for one of a pair of VELA spacecraft, during any single eclipse season.

Nomenclature

a = semimajor axis of spacecraft orbit
b = $(60.0/a)$ sin5.14°; used in calculating γ for the moon-shadow cross section
D = angular parameter related to eclipse duration
f_m = true anomaly of moon, measured from Ω_m
i_ϵ = inclination of spacecraft orbit to ecliptic plane
k = $a/60.0$, ratio of spacecraft orbit semimajor axis to semimajor axis of moon's orbit
n = mean angular motion of spacecraft
$p(\alpha + \eta)$ = probability density function on right ascension
q_γ = probability density function on declination γ
$q(\alpha)$ = conditional probability that $\gamma_1 < \gamma(\alpha) < \gamma_2$
r = probability of occurrence of a moon-shadow eclipse
r_s = radius of conical moon-shadow, at the point in question
Δt = time interval between spacecraft nodal crossing and moon-shadow eclipse
x,y,z = Earth-centered ecliptic coordinate system
z_m = z component of moon's position vector
α = ecliptic right ascension of spacecraft, measured from Ω_ϵ
α_m = ecliptic right ascension of moon, measured from Earth-sun line
γ = declination of moon-shadow cross section
γ_1,γ_2 = declination limits within which an eclipse will occur
η = angle between spacecraft line of nodes and the Earth-sun line
η_m = angle from Earth-sun line to the moon-shadow cross section
Ω_ϵ = ascending node of spacecraft orbit, on ecliptic plane
Ω_m = ascending node of moon orbit, on ecliptic plane

Introduction

A SPACECRAFT powered by solar panels must depend on its battery system for electrical power during an eclipse. The solar panels are used to recharge the batteries in the intervals between successive eclipses. In most cases, batteries have been sized according to the length of the longest Earth-shadow eclipse that could occur on the intended orbit. The possible occurrence of moon-shadow eclipses is usually neglected, because these are usually of shorter duration than the Earth-shadow variety.

However, it is possible to encounter a moon-shadow eclipse shortly after or before an Earth-shadow eclipse. To the spacecraft battery, such an event would be almost equivalent to a continuous eclipse of long duration. Under certain conditions nearly-simultaneous eclipses could lead to excessive battery discharge. Therefore, it is desirable to have a method for estimating the probability of occurrence.

A basic difference in complexity exists between the Earth-shadow and moon-shadow eclipse problems for a spacecraft orbiting about the Earth. An Earth-shadow eclipse involves the shadow of the central body. For orbits of small eccentricity, the spacecraft moves approximately normal to the axis of the central body shadow, thereby reducing the potential duration of the eclipse. The calculation of approximate eclipse durations is relatively simple, and does not require highly-accurate spacecraft ephemeris data. Earth-shadow eclipses of high-altitude spacecraft can only occur during two well-defined times of the year, called eclipse seasons. These two seasons are centered about the dates on which the axis of the Earth-shadow coincides with the spacecraft line of nodes. During any such season the spacecraft will definitely encounter several eclipses, each spaced in time by approximately one orbit period.

However, simple statements of this type cannot be made for moon-shadow eclipses, because of the more complicated geometry of the problem. A spacecraft which encounters a moon-shadow eclipse may move normal to, parallel to, or at 45° to the axis of the shadow, depending on the relative locations of the sun, moon, spacecraft, and the orbit plane of the latter. Because the shadow is not that of the central body, the problem is considerably more complicated than the Earth-shadow case. One new possibility in the moon-shadow case is that the spacecraft may tend to travel down the axis of the shadow, producing an eclipse of several hours duration for high-altitude orbits. Of two orbits which differ only slightly (e.g., in period), one may produce a moon-shadow eclipse of substantial duration, whereas the other produces no eclipse at all. Thus, for a deterministic prediction of moon-shadow eclipses, accurate long-term ephemeris information is required for both the spacecraft and the moon.

An existing computer program[1,2] can be used to determine whether a spacecraft in a specified orbit about the Earth will encounter an eclipse caused by the moon. However, in the early stages of a mission analysis study, possible launches of the spacecraft at many different times on many different dates must be considered. Performing a deterministic moon-shadow eclipse study by investigating each possible combination of launch time and launch date would be

Received December 23, 1968, revision received December 1, 1969.

* Member of the Technical Staff, Mission Analysis Department. Member AIAA.

prohibitively expensive. If the possible dates and times could be reduced to only a few combinations, the cost could be reduced to an acceptable level. However, this only becomes possible relatively late in the mission planning effort. An alternate approach is required for preliminary mission analysis.

Analysis of Eclipse Probabilities

The method that has been adopted here is to assess the probability that a spacecraft in a nominal orbit will encounter a moon-shadow eclipse. Use of the term nominal refers to the fact that the dependence of the precise orbit parameters (e.g., osculating period) on launch time and date has not been taken into account. A convenient feature of the probabilistic approach is that it does not require high-precision ephemeris data for the spacecraft and the moon. If the eclipse probability is shown to be small, the accurate deterministic study can safely be postponed until the nominal launch date(s) has been chosen.

A simple geometrical model of the sun-Earth-moon-spacecraft system will be used to calculate the probability of the spacecraft entering the moon-shadow. The resulting probabilities will be sufficiently accurate for preliminary mission analysis purposes. The moon is assumed to move in a circular orbit of radius 60.0 Earth radii (e.r.), inclined by 5.14° to the ecliptic plane. The spacecraft is assumed to be in a circular orbit with semimajor axis a, inclined by an angle i_ϵ to the ecliptic plane. It will be assumed that the semimajor axis is in the range 40,000–80,000 naut miles.

The axis of the Earth-shadow lies in the ecliptic plane. The axis of the moon-shadow is almost parallel to the ecliptic plane, but may be displaced normal to the plane by as much as 5.4 e.r. Because the sun-moon and sun-Earth distances are so large compared to the Earth-moon distance, the axes of the two shadows are almost parallel, as shown in Fig. 1.

Because the geometrical model is only approximate, the analysis has not been made as detailed as could be done. Various approximations can easily be improved to achieve additional accuracy.

Moon-Shadow Eclipses near a Spacecraft Node

If the spacecraft ecliptic inclination is relatively large, moon-shadow eclipses can only occur in the vicinity of one of the nodes. At other points on its orbit, the spacecraft is too far out of the ecliptic plane to enter the moon-shadow. As seen from above the ecliptic plane, the geometrical situation is shown in Fig. 2. It is convenient to measure angles in the ecliptic plane relative to the Earth-sun line, which is parallel to the axis of the moon-shadow. The direction of the Earth-sun line in inertial space is determined by the day

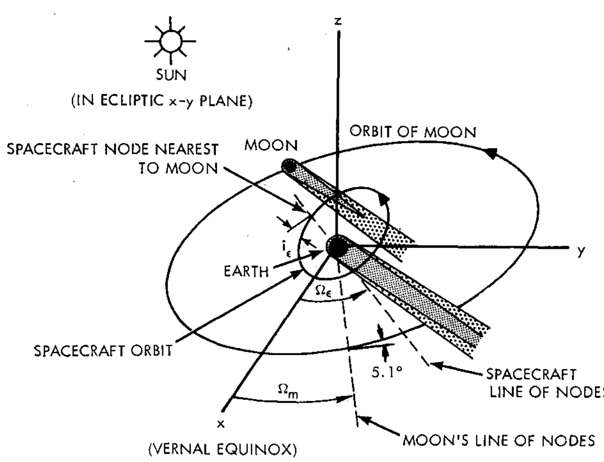

Fig. 1 Geometry of moon-shadow eclipses.

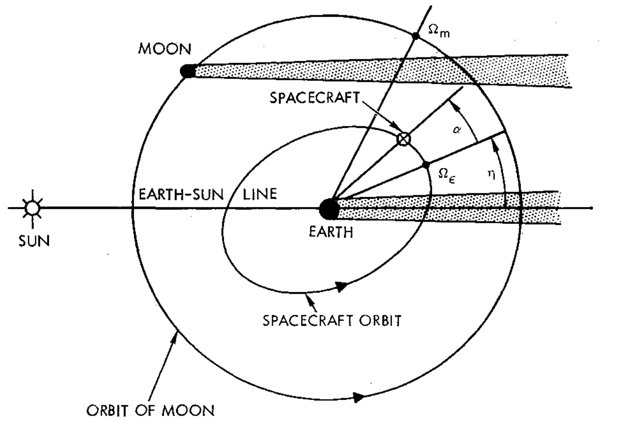

Fig. 2 Geometry as seen from the ecliptic north pole.

of the year being considered. Let η denote the angle between the spacecraft ascending node Ω_ϵ and the Earth-sun line. The value of η specifies the time point in the cycle of Earth-shadow eclipse seasons which is to be considered. However, η by itself does not specify the inertial location of the spacecraft node.

Although the moon-shadow is slightly conical, it may be locally approximated, over the small region where the spacecraft orbit intersects it, as a cylinder of radius r_s. The value of r_s depends on the distance of the intersection from the moon, and hence on η. The spacecraft orbit may be locally approximated as a straight line. A diagram of the intersection between the spacecraft orbit and the moon-shadow is shown in Fig. 3. The shadow cross section, formed by a plane tangent to the spacecraft orbit at a given point near the node, is an ellipse with semimajor axis $[r_s/\cos(\eta + \alpha)]$ and semiminor axis r_s, as shown in Fig. 4. For $(\eta + \alpha)$ near 90° the elliptical cross section becomes more and more elongated. The spacecraft tends to travel down the axis of the shadow, thereby producing eclipses of longer duration.

The moon-shadow oscillates above and below the ecliptic plane during the monthly cycle. Displacement of the shadow cross section from the ecliptic plane causes a slight increase in the semiminor axis and a slight rotation of the principal axes, but to a much smaller amount than is caused by large η. These minor effects will be neglected.

The center of the shadow cross section will be displaced from the ecliptic plane by amount z_m, the z component of the moon's position vector. This out-of-plane displacement may be represented by an angle γ, measured from the center of the Earth, given by

$$\gamma = \arcsin(z_m/a) \tag{1}$$

This angle represents the declination of the center of the moon-shadow cross section, as shown in Fig. 4.

There are times of the lunar month when the moon-shadow cannot possibly intersect the spacecraft orbit. Let α_m denote the right ascension of the moon, measured relative to

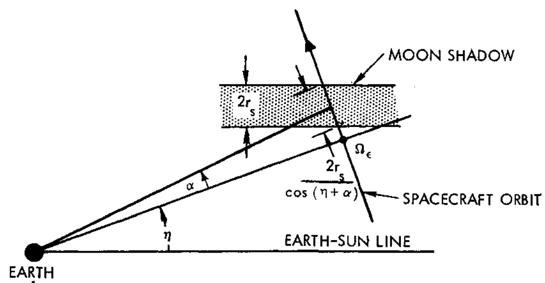

Fig. 3 Intersection of spacecraft orbit with moon shadow.

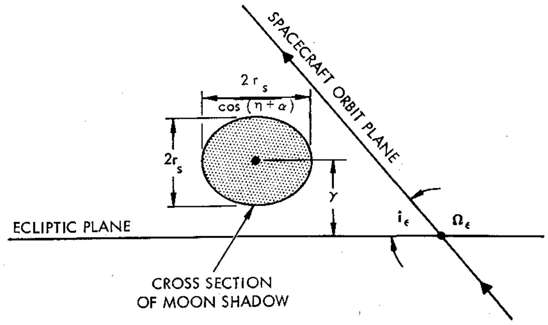

Fig. 4 Moon-shadow cross section near the spacecraft node.

the Earth-sun line as shown in Fig. 5. The moon-shadow cannot intersect the spacecraft orbit unless

$$180° - \arcsin(k) < \alpha_m < 180° + \arcsin(k) \qquad (2)$$

Let η_m represent the angle from the Earth-sun line to the point where the axis of the moon-shadow intersects the spacecraft orbit, as seen from the ecliptic north pole. Because this point will be near the spacecraft line of nodes,

$$\sin\eta_m = (1/k)\sin\alpha_m \qquad (3)$$

If the spacecraft and the center of the cross section both have the same right ascension, the angle α is given by

$$\alpha = \eta_m - \eta = -\eta + \arcsin[(1/k)\sin\alpha_m], \quad -k < \sin\alpha_m < k \qquad (4)$$

Hence, the shadow cross section has ecliptic right ascension $(\eta + \alpha)$ only if α_m is given by

$$\sin\alpha_m = k\sin(\alpha + \eta) \qquad (5)$$

Because $d\eta/dt \cong \dot{\Omega}_\epsilon - 0.98°/\text{day}$, η may be approximated as constant during a single spacecraft orbit. Also, during the few hours when the spacecraft is in the near vicinity of the moon-shadow, α_m can be treated as constant, because $d\alpha_m/dt \cong 0.5°/\text{hr}$. However, between successive nodal crossings of the spacecraft, α_m will change by a non-negligible amount.

Probability Density Function $p(\alpha + \eta)$

The instantaneous true anomalies of the spacecraft and moon, in their respective orbits, will be treated as being statistically independent of one another. This is equivalent to a lack of knowledge of the time of nodal crossing for the spacecraft, or to inaccurate long-term ephemeris data for the relative positions of the two objects. At a random instant

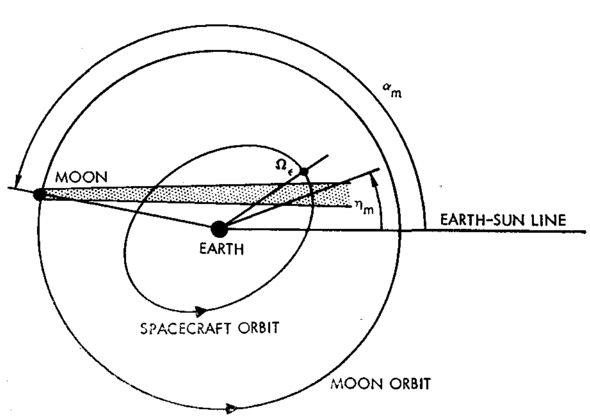

Fig. 5 A lunar position which could produce an eclipse.

the probability that the right ascension of the moon lies between α_m and $\alpha_m + d\alpha_m$ is equal to $d\alpha_m/360°$, because its orbit is assumed circular and its inclination is small.

Consider an instant when the spacecraft, being in the vicinity of its ascending or descending node, has ecliptic right ascension equal to a particular value $(\eta + \alpha)$. It will be convenient to measure right ascensions relative to the Earth-sun line, as shown in Fig. 2. The probability, p, that the center, η_m, of the shadow cross section also lies between $\eta + \alpha$ and $\eta + \alpha + d\alpha$ at the same instant is inversely proportional to $d\eta_m/dt$. From Eq. (4) it may be shown that

$$\frac{d\eta_m}{dt} = \pm\frac{[1 - k^2\sin^2(\alpha + \eta)]^{1/2}}{k\cos(\alpha + \eta)}\frac{d\alpha_m}{dt} \qquad (6)$$

with $d\alpha_m/dt =$ constant. Therefore

$$p = \frac{C\cos(\alpha + \eta)}{[1 - k^2\sin^2(\alpha + \eta)]^{1/2}} \qquad (7)$$

For eclipses at the far spacecraft node, the value of the normalization constant C may be determined from the relation

$$\int_{\alpha_m = \pi - \arcsin(k)}^{\alpha_m = \pi + \arcsin(k)} \frac{d\alpha_m}{360°} = \int_{\alpha+\eta=-\pi/2}^{\alpha+\eta=\pi/2} p(\alpha + \eta)d(\alpha + \eta) \qquad (8)$$

(The limits of integration should be functions of η and i_ϵ, but this will be neglected here for simplicity. Using the accurate limits of integration would reduce the value of C slightly.) Hence,

$$p(\alpha + \eta) = \frac{(k/2\pi)\cos(\alpha + \eta)}{[1 - k^2\sin^2(\alpha + \eta)]^{1/2}} \qquad (9)$$

at the far spacecraft node. A similar procedure can be used to obtain $p(\alpha + \eta)$ at the near node.

Shadow Declinations that Produce an Eclipse

If the moon-shadow center and spacecraft are each at the same right ascension $\eta + \alpha$ there is a range of values of γ which will produce an eclipse in the vicinity of α, as shown in Fig. 6. If $\gamma_1 < \gamma < \gamma_2$ when the spacecraft passes through ecliptic right ascension $\eta + \alpha$, it will be eclipsed somewhere near that point. By calculating where the point of tangency occurs on the elliptical shadow cross section, it may be shown

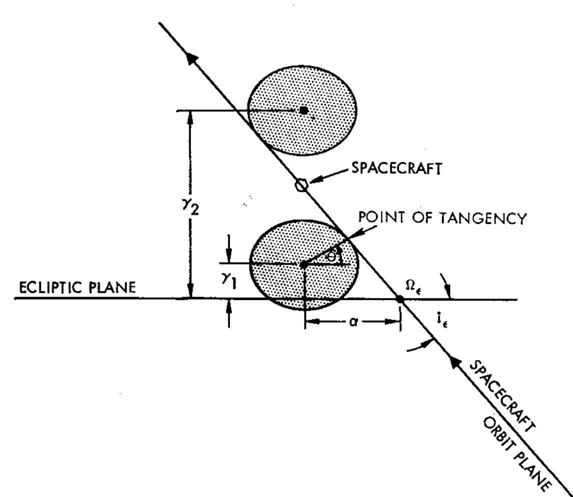

Fig. 6 Range of shadow declinations required for an eclipse.

that

$$\begin{cases} \gamma_1 = [\alpha - r_s \tan(\eta + \alpha)] \tan i_\epsilon - \\ \qquad \dfrac{r_s \cos(\eta + \alpha)(\sin\theta + \tan i_\epsilon \cos\theta)}{1 + \cos\theta \sin(\eta + \alpha)} \\ \gamma_2 = -\gamma_1 + 2\alpha \tan i_\epsilon \end{cases} \tag{10}$$

where

$$\begin{cases} \sin\theta = \sin i_\epsilon \cos i_\epsilon [1 - \cos^2(\eta + \alpha)]^{1/2} + \\ \qquad \cos i_\epsilon [\sin^2 i_\epsilon + \cos^2 i_\epsilon \cos^2(\eta + \alpha)]^{1/2} \\ \cos\theta = [1 - \sin^2\theta]^{1/2} \end{cases} \tag{11}$$

The angle θ, as defined by Eq. (11), should be measured to a line connecting the point of tangency with the focus of the elliptical cross section, rather than the center. However, the adopted approximation is valid for small $(\eta + \alpha)$.

Because the ecliptic inclination of its orbit is only 5.14°, the right ascension of the moon relative to the vernal equinox is approximately equal to $\Omega_m + f_m$, where f_m denotes the true anomaly. The right ascension α_m of the moon at the instant of interest, relative to the Earth-sun line, is given by

$$\alpha_m = (\Omega_m - \Omega_\epsilon) + f_m + \eta \tag{12}$$

The moon's displacement from the ecliptic plane is

$$z_m = ab \sin f_m \tag{13}$$

Thus, having chosen to investigate conditions when $\eta_m = \alpha + \eta$, it follows from Eq. (1) that the declination of the shadow center is

$$\gamma = \arcsin[b \sin(\alpha_m - \eta + \Omega_\epsilon - \Omega_m)] \tag{14}$$

The spacecraft node Ω_ϵ depends on launch time and date, because of the Earth's rotation. Regression of the moon's node causes Ω_m to depend on the date for which the eclipse calculation is to be performed. Hence a large uncertainty in launch time and date for a proposed spacecraft, which always exists during a preliminary eclipse analysis, implies a corresponding uncertainty in $(\Omega_\epsilon - \Omega_m)$. It will be assumed in what follows that $(\Omega_\epsilon - \Omega_m)$ is a random variable uniformly distributed over the range $(0, 2\pi)$. A probability distribution on γ must then be used to calculate the conditional probability, q, that $\gamma_1 < \gamma(\alpha) < \gamma_2$ for the specified α.

Alternately, $(\Omega_\epsilon - \Omega_m)$ could be assumed uniformly distributed over a smaller interval than $(0, 2\pi)$. For example, this might correspond to launch time being restricted to a narrow window on each day. The latter case is somewhat more complex, and will not be treated here.

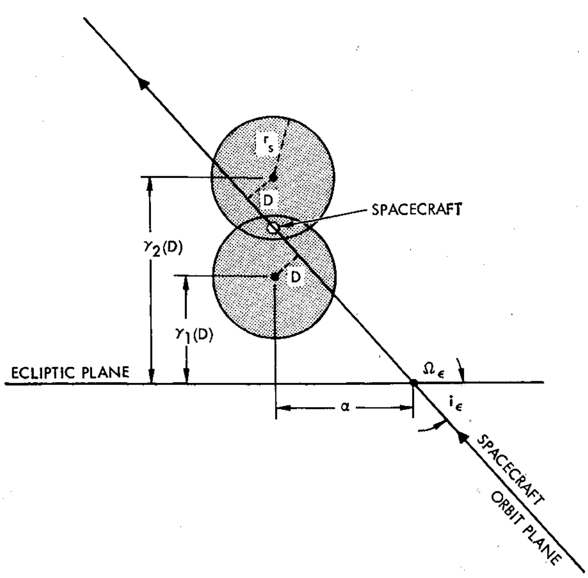

Fig. 7 Geometrical significance of the eclipse duration parameter *D*.

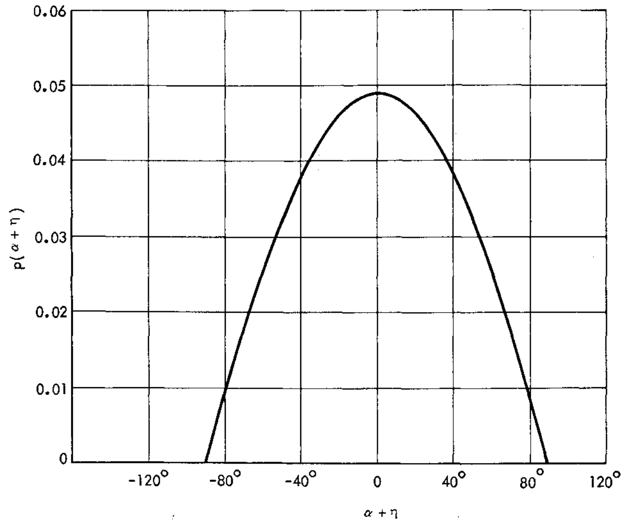

Fig. 8 Probability density $p(\alpha + \eta)$ for VELA orbits.

Probability $q(\alpha)$

For the case where $(\Omega_\epsilon - \Omega_m)$ is uniformly distributed over $(0, 2\pi)$, α does not determine γ. However, α influences γ_1 and γ_2 through Eq. (10). The conditional probability, $q(\alpha)$, that $\gamma_1 < \gamma < \gamma_2$ when the spacecraft and shadow center are both at right ascension $\eta + \alpha$ must be calculated. Using Eqs. (1) and (13),

$$d\gamma/dt = n_m(b^2 - \sin^2\gamma)^{1/2}/\cos\gamma \tag{15}$$

where n_m denotes the mean angular motion of the moon. The probability density, q_γ, that the shadow center is between γ and $\gamma + d\gamma$ at a random instant is inversely proportional to $d\gamma/dt$. Hence

$$q_\gamma = \frac{B \cos\gamma}{(b^2 - \sin^2\gamma)^{1/2}} \tag{16}$$

where B is a normalization constant. The value of B may be determined from

$$1 = \int_{-\sin^{-1}(b)}^{\sin^{-1}(b)} q_\gamma d\gamma = \pi B \tag{17}$$

Therefore

$$q_\gamma = \frac{\cos\gamma}{\pi(b^2 - \sin^2\gamma)^{1/2}}, \quad \sin^2\gamma \le b^2 \tag{18}$$

The functional form of q_γ confirms the intuitive feeling that the moon is likely to be found, at random instant, relatively far from the ecliptic plane.

The conditional probability, $q(\alpha)$, that $\gamma_1 < \gamma < \gamma_2$ at the instant when the spacecraft and shadow center both have right ascension $\eta + \alpha$ is given by

$$q(\alpha) = \int_{\gamma_1}^{\gamma_2} q_\gamma d\gamma = \frac{1}{\pi} \arcsin\left(\frac{\sin\gamma_2}{b}\right) - \frac{1}{\pi} \arcsin\left(\frac{\sin\gamma_1}{b}\right) \tag{19}$$

where $\gamma_1(\alpha)$ and $\gamma_2(\alpha)$ are given by Eq. (10), with $\gamma_2 \le \arcsin(b)$.

Moon-Shadow Eclipse Probability $r(\Delta t)$

The probability of a moon-shadow eclipse between right ascensions α and $\alpha + d\alpha$, relative to the spacecraft node, is

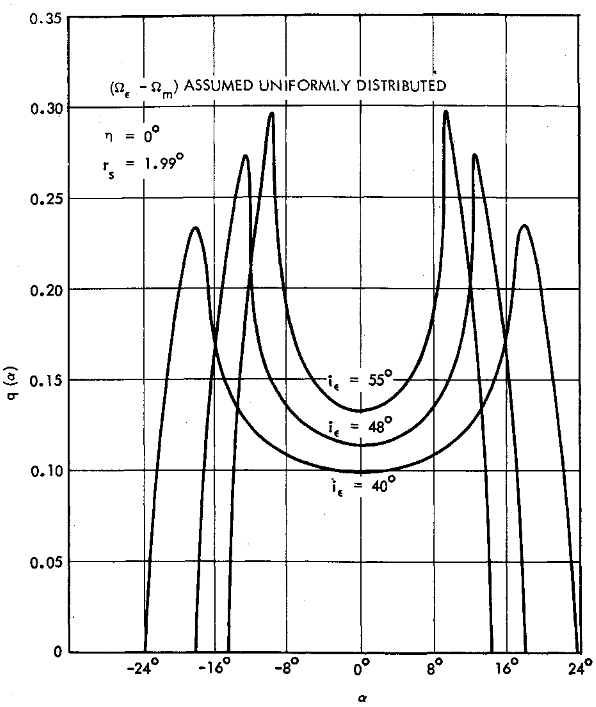

Fig. 9 The probability $q(\alpha)$ for VELA orbits.

equal to $pqd\alpha$. Hence the probability, $r(\alpha^*)$, that an eclipse will occur with $|\alpha| < \alpha^*$ on any single nodal crossing is given by

$$r(\alpha^*) = \int_{-\alpha^*}^{\alpha^*} pqd\alpha \qquad (20)$$

The parameter α^* is a measure of the time interval between the spacecraft nodal crossing and the occurrence of the moon-shadow eclipse. Denoting the mean motion of the spacecraft by n, the time required to go from node to α^* is given by

$$\Delta t \cong \alpha^*/n \cos i_\epsilon \qquad (21)$$

Hence $r(\alpha^*)$, or equivalently $r(\Delta t)$, represents the probability that the spacecraft will enter the moon-shadow within time interval $\pm\Delta t$ of any single nodal crossing.

Eclipses Greater than a Specified Duration

Calculation of the probability of an eclipse longer than a specified duration requires a modification of the previous

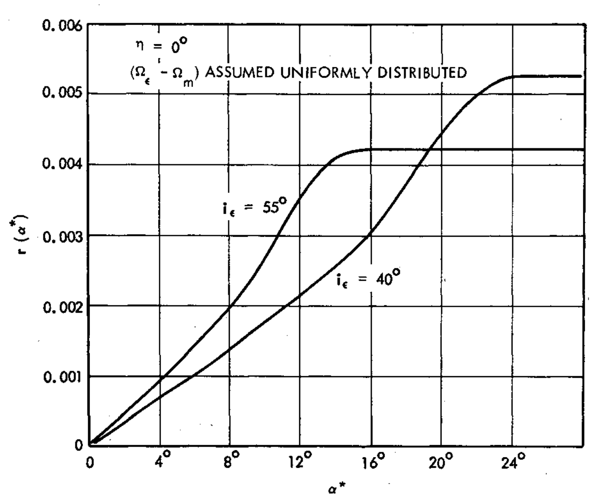

Fig. 10 Moon-shadow eclipse probability at a single VELA nodal crossing.

formulas for γ_1 and γ_2. Only the case $\eta \cong 0°$ will be considered in detail. Hence, the shadow cross section can be treated as circular with radius r_s, as shown in Fig. 7. The arc length along which the spacecraft is within the shadow is equal to $2(r_s^2 - D^2)^{1/2}$. The duration T of the moon-shadow eclipse is given by

$$T = 2(r_s^2 - D^2)^{1/2}/n, \; D \leq r_s \qquad (22)$$

(This equation neglects the slight motion of the moon-shadow during the eclipse.) By properly choosing the numerical value of r_s, both the umbral and penumbral eclipse durations can be treated.

It may be shown that

$$\begin{cases} \gamma_2 - \gamma_1 = \dfrac{2D}{\cos i_\epsilon} \\ \gamma_1 = (\alpha - \dfrac{D}{\sin i_\epsilon})\tan i_\epsilon \end{cases} \qquad (23)$$

If an eclipse occurs with $D \leq D^*$, its duration will be greater than or equal to $2(r_s^2 - D^{2*})^{1/2}/n$. The corresponding probability of occurrence is given by Eqs. (19) and (20), with γ_1 and γ_2 given by Eq. (23). In particular, the maximum eclipse duration is equal to $2r_s/n$, and corresponds to $D = 0$. Eqs. (19, 20, and 23) verify that the probability of an eclipse with duration in excess of $2r_s/n$ is zero.

The numerical probability results to be given in the next section correspond to durations merely greater than zero.

Numerical Results for VELA Orbits

The VELA nuclear detection satellites are injected into nearly-circular orbits at a nominal altitude of 60,000 naut miles. The ecliptic inclination of the orbit is constrained to be greater than about 40°, to reduce the length of the semi-annual Earth-shadow eclipse seasons. Moon-shadow eclipses can only occur in the vicinity of the spacecraft line of nodes.

The function $p(\alpha + \eta)$ is plotted in Fig. 8 for VELA orbits. The decrease to zero for $(\alpha + \eta) = \pm 90°$ is caused by the fact that the shadow center moves very rapidly, in an angular sense as seen from Earth, near the point where it is just tangent to the spacecraft orbit.

The probability $q(\alpha)$ is plotted in Fig. 9 for $\eta = 0°$ and three different values of i_ϵ, assuming $(\Omega_\epsilon - \Omega_m)$ to be uniformly distributed over $(0, 2\pi)$. For a given i_ϵ, q initially increases with $|\alpha|$, illustrating the fact that the shadow center is likely to be relatively far from the ecliptic plane. However, q declines rapidly to zero for some limiting value of $|\alpha|$, corresponding to the VELA spacecraft being too far out of the ecliptic plane to encounter an eclipse.

Sample curves of $r(\alpha^*)$ for a uniformly distributed $(\Omega_\epsilon - \Omega_m)$ are shown in Fig. 10. Figure 10 illustrates that the overall probability of a Moon-shadow eclipse decreases as i_ϵ increases. Eclipses tend to occur closer to the spacecraft node for large i_ϵ. In addition, $r(\alpha^*)$ will depend on η because of the influence of the latter on r_s and $p(\alpha + \eta)$. In Fig. 10 $\eta = 0°$, corresponding to an Earth-shadow eclipse season.

Nearly-Simultaneous Eclipses for VELA

Numerical values of $\Delta t(\alpha^*)$ have been calculated for the VELA spacecraft, using the ideas described above. The data in Fig. 10 can then be used to plot r as a function of Δt. Two such curves are presented in Fig. 11, for $i_\epsilon = 40°$ and 55°. Both curves level off to a constant value, corresponding to the spacecraft getting too far out of the ecliptic plane to encounter the moon-shadow. Time separations of 9.5 hr between moon-shadow eclipse and nodal crossing are possible on VELA orbits.

The numerical results shown in Fig. 11 can be used to assess the probability of occurrence of a moon-shadow eclipse within a few hours of an Earth-shadow eclipse. If this occurs, it must happen during the Earth-shadow eclipse season, and must occur in the vicinity of the VELA node that is farthest from the moon. Only penumbral moon-shadow eclipses are possible at this node, because the umbral shadow does not extend this far. For $i_\epsilon > 40°$, Earth-shadow eclipses must occur within ± 1.5 hr of the VELA nodal crossing. If a moon-shadow eclipse occurs within $\pm \Delta t$ of a nodal crossing which produces an Earth-shadow eclipse, it will be within $\pm(\Delta t + 1.5)$ hours of the Earth-shadow eclipse.

For a given VELA spacecraft, there may be 1, 2, or 3 nodal crossings per season which produce an Earth-shadow eclipse. In most cases there will be two Earth-shadow eclipses per season, for $i_\epsilon > 40°$. Hence, the probability that a particular spacecraft will encounter a moon-shadow eclipse within $\pm \Delta t$ of one of the nodal crossings which produces an Earth-shadow eclipse will be approximately $2r(\Delta t)$. An additional factor of 2 must be introduced, because two VELA spacecraft are launched on the same booster. Thus, the probability that during any single Earth-shadow eclipse season, one of a pair of VELA spacecraft will encounter a moon-shadow eclipse within 11 hours of an Earth-shadow eclipse is approximately 0.021 for $i_\epsilon = 40°$. For $i_\epsilon = 55°$, the probability of two eclipses separated by less than 9.5 hr is 0.017.

Moon-Shadow Eclipses at the Near Spacecraft Node

Moon-shadow eclipses can also occur in the vicinity of the spacecraft node that is nearest to the moon. Such eclipses are of shorter duration and would be spaced by approximately half a VELA period (~55 hours) from any possible Earth-shadow eclipse. (It must be pointed out, however, that short umbral moon-shadow eclipses are possible at this node.) To assess the probability that a moon-shadow eclipse will occur within one VELA period of an Earth-shadow eclipse, the possible occurrence of eclipses in the vicinity of the near node must be included.

The geometry of Fig. 6 is still valid at the VELA node nearest to the moon. The only change is that $r_s = 1.47°$, instead of 1.99°, because the penumbral shadow has not diverged as much. The corresponding probabilities, being approximately proportional to r_s, are somewhat smaller. Curves analogous to Fig. 11, but for the near node, could be drawn. These eclipses would occur half a VELA period, plus or minus a few hours, from the nearest possible Earth-shadow eclipse.

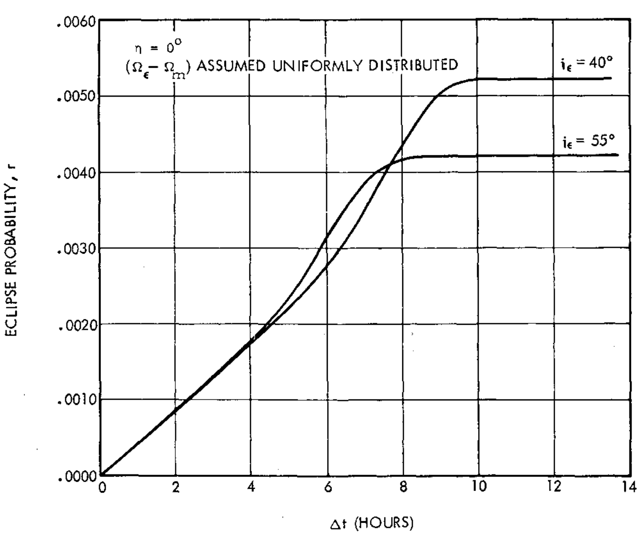

Fig. 11 Moon-shadow eclipse probability at the far VELA node.

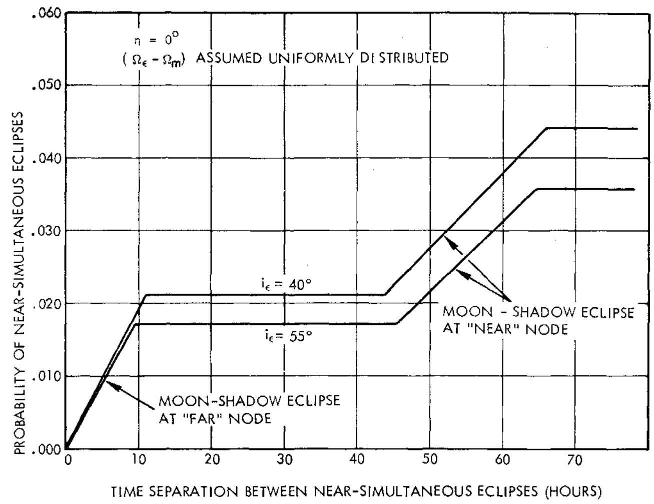

Fig. 12 Probability of near-simultaneous eclipses for a pair of VELA spacecraft.

Figure 12 gives the probability that during any single eclipse season one of a pair of VELA spacecraft will encounter a moon-shadow eclipse within about half a VELA orbit (~55 hr) of an Earth-shadow eclipse. The curves from Fig. 11 have been approximated by straight lines. Each spacecraft will pass the near node three times within half an orbit of Earth-shadow eclipse. (A moon-shadow eclipse at the far node, occurring approximately 110 hours from the nearest Earth-shadow eclipse, would be no more serious than an additional Earth-shadow eclipse.) The probability of near-simultaneous eclipses is seen to be only about 0.044 for $i_\epsilon = 40°$ and 0.036 for $i_\epsilon = 55°$. Knowing that the probability is small, it is reasonable to delay the accurate deterministic moon-shadow eclipse investigation until a nominal launch date and time have been chosen. Figures 10, 11, and 12 do not provide any information about the durations of the moon-shadow eclipses, but this could be generated from the above results.

Summary and Conclusions

It has been shown that moon-shadow eclipses for a proposed high-altitude satellite can be evaluated by use of the probabilistic approach described above. Rather than attempting to answer the question "Will a spacecraft with a specified ephemeris be eclipsed?" the present method addresses the more general question "How probable is an eclipse for a spacecraft in a given type of orbit?" By using the probabilistic approach, the need for highly accurate long term spacecraft and lunar ephemeris data has been avoided. The results can be applied to both umbral and penumbral moon-shadow eclipses having durations greater than a given amount.

The present analysis has centered around the occurrence of eclipses in the vicinity of one of the spacecraft nodes, because of an interest in nearly-simultaneous eclipses for the VELA spacecraft. However, these same ideas can be extended to the treatment of moon-shadow eclipses which occur well away from the spacecraft line of nodes. An analysis could be made of long (several hours) duration eclipses, for high-altitude orbits with low inclination. These would correspond to $(\eta + \alpha) \cong 90°$ in Fig. 2, with the spacecraft tending to travel down the axis of the moon-shadow.

References

[1] Williams, R. R., "Lunar Eclipse Calculations for an Orbiting Spacecraft," 05069-6021-R000, Jan. 1967, TRW Systems, Redondo Beach, Calif.

[2] Escobal, P. R. and Robertson, R. A., "Lunar Eclipse of a Satellite of the Earth," *Journal of Spacecraft and Rockets*, Vol. 4, No. 4, April 1967, pp. 538–540.